LEDUC-RIGHI EFFECT IN SUPERCONDUCTORS WITH NONTRIVIAL DENSITY OF STATES.

N.K.Fedorov
P.N.Lebedev Physical Institute, Russian Academy of Sciences, Moscow 117924, Russia

arXiv:cond-mat/9708203v3 [cond-mat.supr-con] 9 Sep 1997

Increasing of electronic thermal conductivity of superconductor in the model with nontrivial density of states was considered. The electronic thermal conductivity $\kappa(T)$ of SC in the presence of external magnetic field was investigated. It was shown that if symmetrical part of quasiparticle scattering rate less than cyclotron energy observed mechanism of increasing $\kappa(T)$ can be separated from connected with symmetrical electronic scattering effects another one.

Introduction. There are several ways to provide the nature of thermal conductivity peak at temperature below T_C in HTSC [1,2].

Connected with temperature dependence of phonon mean free path increasing of lattice thermal conductivity was theoretically predicted in papers [3]. Later this approach was applied to explain the thermal properties of HTSC [4]. But measurements of highly anysotropic thermal current [5] give evidence that such method cannot be exhaustive.

Scattering of electrons by nonmagnetic impurities in anysotropic heavy fermions SC [6] and inelastic scattering of electrons by antiferromagnetic spin fluctuations [7,8] were taken into account to understand the existence of thermal conductivity peak.

In present paper the possible experimental evidence of not connected with scattering effects mechanisms creating of thermal conductivity peak is investigated. As the example of such mechanism can be considered increasing of electronic thermal conductivity of superconductor in the model with nontrivial density of states [9].

Thermal conductivity. The electronic part of thermal conductivity can be derived by kinetic equation for density matrix in nonequilibrium technique of Green-Keldysh functions. It can be shown that the charge current

$$\frac{1}{e}\mathbf{j}_R(T) = \langle\sigma_s(p)\rangle_p(\frac{2e}{\hbar c}\mathbf{A}_\perp - \nabla_R\chi(R)) - \langle\eta_p\rangle_p\nabla_R T \tag{1}$$

and thermal current

$$\mathbf{q}_R(T) = \langle\xi_p\sigma_s(p)\rangle_p(\frac{2e}{\hbar c}\mathbf{A}_\perp - \nabla_R\chi(R)) - \langle\xi_p\eta_p\rangle_p\nabla_R T \tag{2}$$

in superconductor, where $\mathbf{A}_\perp$ - vector-potential of electromagnetic field, $\chi(R)$ - order parameters phase.

Under the condition

$$\mathbf{j}_R(T) = 0 \tag{3}$$

one can obtain from Eqs.(1,2)

$$\mathbf{q}_R(T) = -\kappa(T)\nabla_R T, \tag{4}$$

where

$$\kappa(T) = \langle\xi_p\eta_p\rangle_p - \langle\eta_p\rangle_p\frac{\langle\xi_p\sigma_s(p)\rangle_p}{\langle\sigma_s(p)\rangle_p} \tag{5}$$

is the thermal conductivity coefficient.

Eq.(5) without the second term is the Wiedemann-Franz law. Coefficient $\langle\xi_p\eta_p\rangle_p$ corresponds to quasiparticle current from hot edge of superconductor to cold edge.

To influence to the kinetic properties of SC it is possible to consider superconductor with nontrivial DOS function:

$$N_\xi = N_0 + \left\{\begin{array}{lll} N_1, & if & e_1 \le \xi_k \le e_2, \\ N_2, & if & e_3 \le \xi_k \le e_4; \end{array}\right. \tag{6}$$

which is consists of narrow peak N_1 not far from e_F with center on e_0 and the wide one N_2. The assumption $e_i - \mu(T_C) < 0$ will be taken into account. $d-wave$ symmetry of order parameter is suggested by the $\Delta(k) = \Delta\cos(2\phi)$, where ϕ - angle from $X - axes$ in Brilluoin zone. Parameters of the model can be chosen to provide the condition $\mu(0) - \mu(T_c) \sim T_c$. Then it is possible to obtain [9] the peak of electronic thermal conductivity at $T^* \sim |e_0 - \mu| < T_c$. At this temperature given the main contribution to the thermal conductivity electron-hole excitations have energy $\sim e_0 - \mu(T, \Delta)$.

The back current of condensate is taken into account by the second term Eq.(5). Consideration of this term in linear approximation could be incorrect at temperatures near T_c because of the infinity of $\mathbf{v}_s = (\frac{2e}{\hbar c}\mathbf{A}_\perp - \nabla_R \chi(R))$ under the conditions Eq.(3).

Lets consider the case of electronic conductivity $\frac{\partial J_F}{\partial e_F} > 0$, where J_F is the integral over the isoenergy surface in k-space near e_F. If the DOS has the narrow peak then moving of $\mu(T)$ leads to decreasing of thermoelectric coefficient $\langle\eta\rangle_p$ at temperature $T^* \sim |e_0 - \mu(T^*)|$ in accordance with increasing of hole excitations with energy $\sim T^*$. The same cause can decreases condensate carried heat flow $\langle\xi_p\sigma_S\rangle_p$ can at the temperature $\sim T^*$ Therefore besides the coefficient $\langle\eta\rangle_p \frac{\langle\xi_p\sigma_s\rangle_p}{\langle\sigma_S\rangle_p}$ is always positive the relative value of thermal conductivity peak $\frac{\kappa(T^*)}{\kappa(T_C)}$ increases with comparison to Wiedemann-Franz low derived one. Described mechanism leading to the peak of thermal conductivity could be suppressed by exponential decreasing of quasiparticle quantity in SC with isotropic or had not nods order parameter.

In the case of the hole conductivity $\left(\frac{\partial J_F}{\partial e_F} < 0\right)$ the condensate current decreases peak of electronics thermal conductivity derived by Wiedemann-Franz low for chosen DOS .

Leduc-Righi effect. The experimental evidence of electronic nature of thermal conductivity peak could be Leduc-Righi effect [11]. In SC in external magnetic field $\mathbf{B} \perp \nabla_R T$ (mixed state) an additional thermal current occurs. In this case charge current:

$$\frac{1}{e}\mathbf{j}_R(T) = <\sigma_S>_\parallel \mathbf{v}_S - <\eta>_\parallel \nabla_R T + <\sigma_S>_\perp [\mathbf{b}\times\mathbf{v}_S] - <\eta>_\perp [\mathbf{b}\times\nabla_R T] \tag{7}$$

and thermal current:

$$\mathbf{q}_R(T) = <\xi\sigma_S>_\parallel \mathbf{v}_S - <\xi\eta>_\parallel \nabla_R T + <\xi\sigma_S>_\perp [\mathbf{b}\times\mathbf{v}_S] - <\xi\eta>_\perp [\mathbf{b}\times\nabla_R T], \tag{8}$$

where $\mathbf{b} = \frac{\mathbf{B}}{B}$.

Eq.(7) under condition Eq.(3) gives formula for condensate velocity:

$$v_s = \frac{\left\{\frac{<\eta_p>_{p\parallel}}{<\sigma_s(p)>_{p\parallel}} + \frac{<\eta_p>_{p\perp}}{<\sigma_s(p)>_{p\parallel}}\frac{<\sigma_s(p)>_{p\perp}}{<\sigma_s(p)>_{p\parallel}}\right\}}{1+\left(\frac{<\eta_p>_{p\perp}}{<\sigma_s(p)>_{p\parallel}}\right)^2}\nabla_R T + \frac{\left\{\frac{<\eta_p>_{p\perp}}{<\sigma_s(p)>_{p\parallel}} - \frac{<\eta_p>_{p\parallel}}{<\sigma_s(p)>_{p\parallel}}\frac{<\sigma_s(p)>_{p\perp}}{<\sigma_s(p)>_{p\parallel}}\right\}}{1+\left(\frac{<\eta_p>_{p\perp}}{<\sigma_s(p)>_{p\parallel}}\right)^2}[b\times\nabla_R T] \tag{9}$$

From Eqs.(8,9) it is easy to derive followings expressions for thermal current and thermal conductivity coefficients:

$$\begin{aligned}\mathbf{q} = &-\left\{<\xi\eta>_\parallel - \frac{<\xi\sigma_S>_\parallel}{<\sigma_S>_\parallel}<\eta>_\parallel\right\}\nabla_R T - \left\{<\xi\eta>_\perp - \frac{<\xi\sigma_S>_\parallel}{<\sigma_S>_\parallel}<\eta>_\perp\right\}[\mathbf{b}\times\nabla_R T] \\ &+\left\{<\xi\sigma_S>_\perp - \frac{<\xi\sigma_S>_\parallel}{<\sigma_S>_\parallel}<\sigma_S>_\perp\right\}\frac{<\eta>_\parallel}{<\sigma_S>_\parallel}\times[\mathbf{b}\times\nabla_R T] \equiv -\kappa_\parallel\nabla_R T - \kappa_\perp[\mathbf{b}\times\nabla_R T].\end{aligned} \tag{10}$$

To derive coefficients $\kappa_\parallel, \kappa_\perp$ the kinetic equation for SC in external magnetic field $\mathbf{B}$ can be used:

$$\begin{aligned}&\frac{\partial}{\partial t}\widehat{\rho}_0 + \frac{1}{2}\left\{\left[\frac{\partial}{\partial\mathbf{p}}\hat{e}_p;\frac{\partial}{\partial\mathbf{R}}\widehat{\rho}_0\right]_+ - \left[\frac{\partial}{\partial\mathbf{R}}\hat{e}_p;\frac{\partial}{\partial\mathbf{p}}\widehat{\rho}_0\right]_+\right\} + i\left[\hat{e}_p;\widehat{\rho}_0\right]_- = \\ &= i\omega\,\overset{\wedge}{\delta\rho} - i\left[\hat{e}_p;\overset{\wedge}{\delta\rho}\right]_- + \overset{\wedge}{\sigma}_Z\,\frac{e}{c}(\left[\frac{\mathbf{p}}{m}\times rot\mathbf{A}_f\right]\frac{\partial}{\partial\mathbf{p}})\,\overset{\wedge}{\delta\rho} - \overset{\wedge}{\Sigma}_T\overset{\wedge}{\delta\rho},\end{aligned} \tag{11}$$

where $\widehat{\rho}_0$ - equilibrium density matrix, $\overset{\wedge}{\delta\rho}$ - nonequilibrium correction to $\widehat{\rho}_0$, $\hat{e}_p = \hat{e}\,(p \pm \frac{e}{c}A_f)$ - energy matrix, $\overset{\wedge}{\Sigma}_T$ - relaxation energy matrix, $\mathbf{A}_f$ - vector potential of electromagnetic field in SC. The solution of Eq.(11) gives kinetic coefficients [9,10]:

$$\langle \eta_p \rangle_{p\parallel} = \int \frac{d^3\mathbf{p}}{3m(2\pi\hbar)^3} \frac{\upsilon_\parallel}{\upsilon_\parallel^2 + \upsilon_\perp^2} (\xi_p - \mu) \frac{\partial G(\xi_p - \mu, T, \triangle(T,p))}{\partial T} \tag{12 a}$$

$$\langle \xi_p \eta_p \rangle_{p\parallel} = \int \frac{d^3\mathbf{p}}{3m(2\pi\hbar)^3} \frac{\upsilon_\parallel}{\upsilon_\parallel^2 + \upsilon_\perp^2} (\xi_p - \mu)^2 \frac{\partial G(\xi_p - \mu, T, \triangle(T,p))}{\partial T} \tag{12 b}$$

$$\langle \eta_p \rangle_{p\perp} = \int \frac{d^3\mathbf{p}}{3m(2\pi\hbar)^3} \frac{\upsilon_\perp}{\upsilon_\parallel^2 + \upsilon_\perp^2} (\xi_p - \mu) \frac{\partial G(\xi_p - \mu, T, \triangle(T,p))}{\partial T} \tag{12 c}$$

$$\langle \xi_p \eta_p \rangle_{p\perp} = \int \frac{d^3\mathbf{p}}{3m(2\pi\hbar)^3} \frac{\upsilon_\perp}{\upsilon_\parallel^2 + \upsilon_\perp^2} (\xi_p - \mu)^2 \frac{\partial G(\xi_p - \mu, T, \triangle(T,p))}{\partial T} \tag{12 d}$$

where relaxation rate $\upsilon_p = \upsilon_0 + \upsilon_\perp$. υ_0 corresponds to the symmetrical scattering of electrons, $\upsilon_\perp = v_F \sigma_V \frac{m}{\hbar} \omega_B$; σ_V is effective temperature dependent transport cross-section, correspondent to asymmetric scattering from vortices. $v_F \sigma_V \frac{m}{\hbar} \to 1$ $(T \to T_C)$.

$$G(\xi_k - \mu, T) = \frac{1}{2} - \frac{1}{2} \frac{\xi_k - \mu}{\sqrt{(\xi_k - \mu)^2 + \Delta(T)^2}} \times \tanh\left(\frac{\sqrt{(\xi_k - \mu)^2 + \Delta(T)^2}}{2T} \right);$$
$$F(\xi_k - \mu, T) = \frac{1}{2} \frac{\Delta(T)}{\sqrt{(\xi_k - \mu)^2 + \Delta(T)^2}} \times \tanh\left(\frac{\sqrt{(\xi_k - \mu)^2 + \Delta(T)^2}}{2T} \right)$$

are integrated over frequency ω Green-Keldysh functions G^{-+}, F^{-+}.

Correspondent superconducting kinetics coefficients:

$$\begin{aligned} \langle \sigma_s(p) \rangle_{p\parallel} = \frac{1}{m} \int \frac{d^3\mathbf{p}}{m(2\pi\hbar)^3} \mathbf{p}^2 \frac{2(\xi - \mu)\left(4(\xi - \mu)^2 + \upsilon_\parallel^2 - \upsilon_\perp^2\right)}{\left(4(\xi - \mu)^2 + \upsilon_\parallel^2 - \upsilon_\perp^2\right)^2 + 4\upsilon_\parallel^2 \upsilon_\perp^2} \\ \times \frac{\partial F(\xi_p - \mu, T, \triangle(T,p))}{\partial \xi_p} \Delta(T,p) \end{aligned} \tag{13 a}$$

$$\begin{aligned} \langle \xi_p \sigma_s(p) \rangle_{p\parallel} = \frac{1}{m} \int \frac{d^3\mathbf{p}}{m(2\pi\hbar)^3} \mathbf{p}^2 \frac{2(\xi - \mu)\left(4(\xi - \mu)^2 + \upsilon_\parallel^2 - \upsilon_\perp^2\right)}{\left(4(\xi - \mu)^2 + \upsilon_\parallel^2 - \upsilon_\perp^2\right)^2 + 4\upsilon_\parallel^2 \upsilon_\perp^2} (\xi_p - \mu) \\ \times \frac{\partial F(\xi_p - \mu, T, \triangle(T,p))}{\partial \xi_p} \Delta(T,p) \end{aligned} \tag{13 b}$$

$$\begin{aligned} \langle \sigma_s(p) \rangle_{p\perp} = \frac{1}{m} \int \frac{d^3\mathbf{p}}{m(2\pi\hbar)^3} \mathbf{p}^2 \frac{4(\xi - \mu)\upsilon_\parallel \upsilon_\perp}{\left(4(\xi - \mu)^2 + \upsilon_\parallel^2 - \upsilon_\perp^2\right)^2 + 4\upsilon_\parallel^2 \upsilon_\perp^2} \\ \times \frac{\partial F(\xi_p - \mu, T, \triangle(T,p))}{\partial \xi_p} \Delta(T,p) \end{aligned} \tag{13 c}$$

$$\begin{aligned} \langle \xi_p \sigma_s(p) \rangle_{p\perp} = \frac{1}{m} \int \frac{d^3\mathbf{p}}{m(2\pi\hbar)^3} \mathbf{p}^2 \frac{4(\xi - \mu)\upsilon_\parallel \upsilon_\perp}{\left(4(\xi - \mu)^2 + \upsilon_\parallel^2 - \upsilon_\perp^2\right)^2 + 4\upsilon_\parallel^2 \upsilon_\perp^2} (\xi_p - \mu) \\ \times \frac{\partial F(\xi_p - \mu, T, \triangle(T,p))}{\partial \xi_p} \Delta(T,p) \end{aligned} \tag{13 d}$$

In paper [11] following equations for relaxation rates were used [12]: $\upsilon_\parallel = \frac{v_F}{l_0} + v_F \sigma_{tr} \frac{m}{\hbar} \omega_B$, $\upsilon_\perp = v_F \sigma_V \frac{m}{\hbar} \omega_B$. Here is suggested that $v_F \sim 10^5 m/c$, $\sigma_{tr} \sim 10^{-8} m$, $l_0 \sim 10^{-7} m$, $B \sim 10T$. After this it is possible to rewrite in Leduc-Righi coefficient $\langle \xi_p \eta_p \rangle_{p\perp}$ (assumption that $\frac{\sigma_{tr}}{\sigma_V}$ has the T-independent value is taken into account [11]):

$$\frac{\upsilon_\perp}{\upsilon_\parallel^2+\upsilon_\perp^2} \sim \frac{1}{\upsilon_\perp} - \frac{1}{\upsilon_\perp}\frac{\upsilon_\parallel^2}{\upsilon_\perp^2} \stackrel{(T\to T_C)}{\longrightarrow} \frac{1}{\omega_{_B}} - \frac{1}{\omega_{_B}}\frac{\upsilon_0^2}{\omega_B^2} \tag{14}$$

First term of right side Eq.(14) corresponds to asymmetric scattering of electrons from vortices (Lorenz force in normal state). Second term ($\frac{\upsilon_\parallel}{\omega_B} \sim \frac{\upsilon_0}{\omega_B} \sim 10^{-1}$) is connected with electronic scattering. Presented estimates shows that in pure SC ($\frac{\upsilon_0}{\omega_B} \ll 1$) with short enough transport cross section σ_{tr} it is possible to neglect in $\kappa_\perp$ relaxation rate $\upsilon_\parallel$ (and T-dependent l_0) in comparison to cyclotron energy $\omega_B(\upsilon_\perp \sim 1 \div 10^{-1} \times \omega_B)$.

Conclusions. The temperature dependence of chemicals potential in the model with nontrivial DOS can leads to the peak of electronic thermal conductivity of superconductor.

The possible experimental evidence of the observed mechanism creating of thermal conductivity peak could be connected with measurements of perpendicular to temperature gradient thermal current in magnetic field.

Acknowledgments. Author acknowledges useful discussions with P.I.Arseyev and B.A.Volkov.

This research is supported by Program "Superconductivity" Grant N96-081 .

---

[1] M.F.Crommie, A. Zettl, Phys. Rev. **B43**, 2520, (1991).
[2] S.T. Ting, P.Peranmbuco-Wise, J.E.Crow, Phys. Rev. **B50**, 6375, (1994).
[3] J. Bardeen, G. Rickayzen, L.Tewordt, Phys. Rev. **113**, 982, (1959).
[4] L.Tewordt, Th. Wolkhausen, Solid State Commun. **70**, 839, (1989).
[5] R. Gagnon, S. Pu, B. Ellman, L. Taillefer, Phys. Rev. Lett. **78**, 1976, (1997).
[6] B. Arfi , C.J.Pethick, Phys. Rev. **B38**, 2318, (1988).
[7] P.J. Hirshfeld, W.O. Putikka, D. Scalapino, Phys. Rev. Lett. **71**, 3705, (1993).
[8] P.J. Hirshfeld, W.O. Putikka, D. Scalapino, Phys. Rev. **B50**, 10250, (1994).
[9] N.K.Fedorov, to be published.
[10] P.I.Arseyev, N.K.Fedorov, to be published.
[11] K.Krishana, J.M.Harris, N.P.Ong, Phys. Rev. Lett. **75**, 3529, (1995).
[12] Robert M. Cleary, Phys. Rev. **175**, 578, (1968).